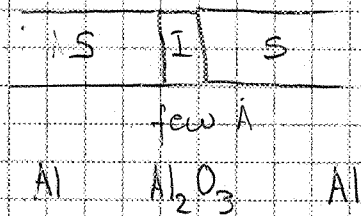
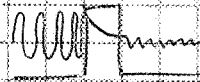


Josephson Effect



tunnel effect



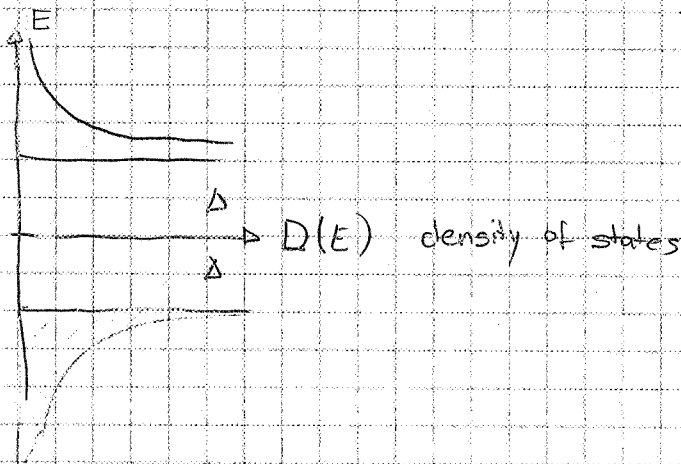
$$e^{-\kappa x} \quad \kappa = \frac{1}{\hbar} \sqrt{2m(V_0 - E)}$$

superconductivity

Bardeen Cooper Schrieffer

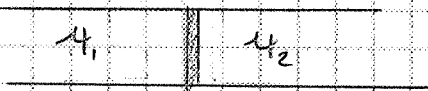
Cooper pairs $2e$ $(\vec{k}, -\vec{k})$

condensate $\Psi = \sqrt{n} e^{i\theta}$



superconductivity class
Zehetmayer / Weber

DC Josephson Effect



$$i\hbar \frac{\partial}{\partial t} \psi_1 = \hbar T \psi_2$$

$$i\hbar \frac{\partial}{\partial t} \psi_2 = \hbar T \psi_1$$

$$\psi_1 = \sqrt{n_1} e^{i\theta_1}$$

$$\psi_2 = \sqrt{n_2} e^{i\theta_2} \quad n_i, \theta_i \in \mathbb{R}$$

$$\frac{\partial}{\partial t} \psi_1 = \frac{1}{2} n_1^{-1/2} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i n_1^{1/2} e^{i\theta_1} \frac{\partial \theta_1}{\partial t} = -i T \psi_2$$

$$\frac{\partial}{\partial t} \psi_2 = \frac{1}{2} n_2^{-1/2} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i n_2^{1/2} e^{i\theta_2} \frac{\partial \theta_2}{\partial t} = -i T \psi_1$$

multiply by $n_1^{1/2} e^{-i\theta_1}$

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -i T (n_1 n_2)^{1/2} e^{i\delta} \quad \delta = \theta_2 - \theta_1$$

$$\frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -i T (n_1 n_2)^{1/2} e^{-i\delta}$$

real part

imaginary part

$$\frac{\partial n_1}{\partial t} = 2T (n_1 n_2)^{1/2} \sin \delta$$

$$\frac{\partial \theta_1}{\partial t} = -T \left(\frac{n_2}{n_1} \right)^{1/2} \cos \delta$$

$$\frac{\partial n_2}{\partial t} = -2T (n_1 n_2)^{1/2} \sin \delta$$

$$\frac{\partial \theta_2}{\partial t} = -T \left(\frac{n_1}{n_2} \right)^{1/2} \cos \delta$$

equal superconductors

$$n_1 \cong n_2$$

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \quad \frac{\partial}{\partial t} \delta = 0$$

$$\frac{\partial n_2}{\partial t} = - \frac{\partial n_1}{\partial t}$$

L6.1

$$I = 2e \frac{\Delta n_1}{\Delta t}$$

$$I = I_0 \sin(\delta) = I_0 \sin(\theta_2 - \theta_1)$$

DC Josephson effect

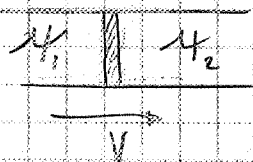
DC supercurrent without voltage

I_0 junction property

\propto exp. thickness

$\propto A$

AC Josephson effect



$$\hbar \hbar \frac{\partial}{\partial t} \psi_1 = \hbar T \psi_2 - eV \psi_1$$

$$i \hbar \frac{\partial}{\partial t} \psi_2 = \hbar T \psi_1 + eV \psi_2$$

$$\frac{1}{2} \frac{\Delta n_1}{\Delta t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{i}{\hbar} eV n_1 - i T (n_1 n_2)^{1/2} e^{i\delta}$$

$$\frac{\Delta n_1}{\Delta t} = 2 T (n_1 n_2)^{1/2} \sin(\delta)$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T (n_1 n_2)^{1/2} \cos \delta$$

$$\boxed{\frac{\partial}{\partial t} (\theta_2 - \theta_1) = \frac{\partial}{\partial t} \delta = 2eV/\hbar}$$

AC Josephson effect

$$I = I_0 \sin\left(\delta(t) - \frac{2eV}{\hbar} t\right)$$

$$\omega = \frac{2eV}{\hbar}$$

$$\hbar\omega = 2eV$$

Cooper pairs

Voltage standard

$$\nu = \frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 10^{-6}}{6.6 \cdot 10^{-34}} \approx \frac{1}{2} \cdot 10^9 \approx 500 \text{ MHz}$$

$$V = \frac{\phi_0}{2\pi} \dot{\delta}$$

$$\phi_0 = \frac{h}{2e} \text{ superconducting flux quantum}$$

$$= 2 \cdot 10^{-5} \text{ Wb} = 20 \text{ Gauss } (\mu\text{m})^2$$

$$\text{Wb} = \text{Tesla} \cdot \text{m}^2$$

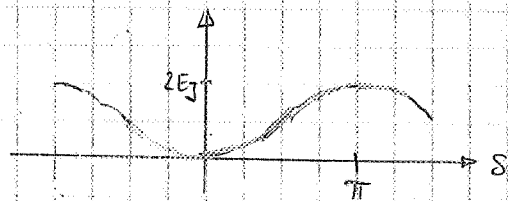
Josephson energy

$$E(\delta) = \int_0^z IV \, dz = \frac{I_0 \phi_0}{2\pi} \int_0^z \sin(\mu) \frac{d\mu}{dz} \, dz$$

$$= \frac{I_0 \phi_0}{2\pi} \int_0^\delta \sin(\mu) \, d\mu = \frac{I_0 \phi_0}{2\pi} (1 - \cos(\delta))$$

$$E = E_J (1 - \cos(\delta))$$

$$E_J = \frac{I_0 \phi_0}{2\pi}$$



Josephson inductance

$$L = \frac{V}{\dot{I}} = \frac{\phi_0 / 2\pi \dot{\delta}}{I_0 \frac{d}{dt} (\sin(\delta))} = \frac{\phi_0}{2\pi I_0} \frac{\frac{d\delta}{dt}}{\cos(\delta) \frac{d\delta}{dt}}$$

$$L = \frac{\phi_0}{2\pi I_0} \frac{1}{\cos(\delta)}$$

Lb.3

gauge change & phase

Hamiltonian for a charged particle
in a EM field

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\Phi$$

physical fields

$$\vec{E} = -\vec{\nabla}\phi - \frac{d\vec{A}}{dt}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

change gauge

scalar field $\chi(\vec{x}, t)$

unit $[\chi] = \text{Volt} \cdot \text{sec}$

$$\vec{A} \longrightarrow \vec{A}' = \vec{A} + \vec{\nabla}\chi$$

$$\phi \longrightarrow \phi' = \phi - \frac{d\chi}{dt}$$

$$\psi \longrightarrow \psi' = \psi \cdot e^{i\chi q/\hbar}$$

gauge invariant phase difference

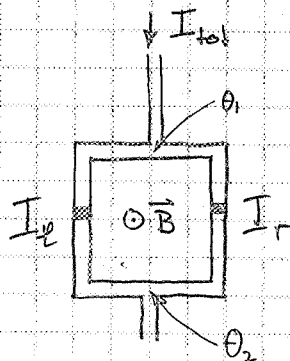
$$\delta = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0} \int_1^2 \vec{A} \cdot d\vec{\ell}$$

$$\begin{aligned} \delta \longrightarrow \delta' &= \theta_2 + \chi(\vec{x}_2, t) \frac{2e}{\hbar} - \theta_1 - \chi(\vec{x}_1, t) \frac{2e}{\hbar} \\ &\quad - \frac{2\pi}{\phi_0} \int_1^2 (\vec{A} + \vec{\nabla}\chi) \cdot d\vec{\ell} = \delta \end{aligned}$$

$$\text{using } \frac{2\pi}{\phi_0} \int_1^2 \vec{\nabla}\chi \cdot d\vec{\ell} = \frac{2\pi}{\phi_0} \chi \Big|_1^2 = \frac{2\pi}{\phi_0} (\chi(\vec{x}_2) - \chi(\vec{x}_1))$$

SQUID - superconducting quantum interference device

L6.4



$$I_{\text{total}} = I_l + I_r = I_0 \sin(\delta_l + I_0 \sin|\delta_r|)$$

assumed equal junctions

$$\delta_l = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0} \int_{\text{left path}} \vec{A} \cdot d\vec{l}$$

$$\delta_r = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0} \int_{\text{right path}} \vec{A} \cdot d\vec{l}$$

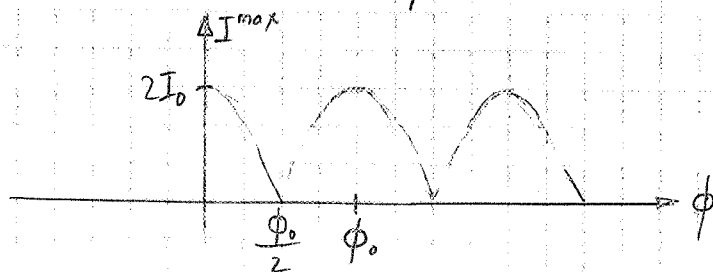
$$I_{\text{total}} = 2I_0 \sin\left(\frac{\delta_l + \delta_r}{2}\right) \cos\left(\frac{\delta_l - \delta_r}{2}\right)$$

$$= 2I_0 \sin\left(\frac{\delta_l + \delta_r}{2}\right) \cos\left(\frac{\pi}{\phi_0} \oint \vec{A} \cdot d\vec{l}\right)$$

using $\oint \vec{A} \cdot d\vec{l} = \int_S \nabla \times \vec{A} \cdot d\vec{\sigma} = \int_S \vec{B} \cdot d\vec{\sigma} = \phi$

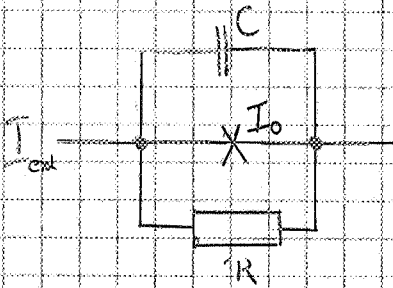
maximal supercurrent, i.e. switching current

$$I_{\text{total}}^{\text{max}} = 2I_0 \left| \cos\left(\pi \frac{\phi}{\phi_0}\right) \right|$$



L6.5

RCSJ Model (resistively and capacitively shunted junction)



$$I = I_0 \sin(\delta) + \frac{V}{R} + C \frac{dV}{dt} \quad \left(\begin{array}{l} Q = CV \\ I = \dot{Q} \end{array} \right)$$

$$V = \frac{\phi_0}{2\pi} \dot{\delta}$$

$$I = I_0 \sin(\delta) + \frac{\phi_0}{2\pi R} \dot{\delta} + \frac{C \phi_0}{2\pi} \ddot{\delta}$$

$$z = \omega_p z$$

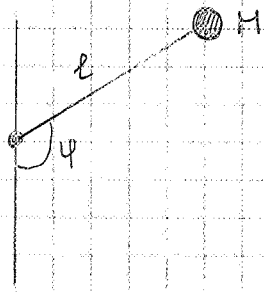
$$\omega_p = \sqrt{\frac{I_0^2}{C \phi_0}} = \sqrt{\frac{I_0^2 e^2}{\hbar C}} \quad \text{plasma frequency}$$

$$Q = \omega_p RC$$

$$\beta_c = Q^2 \quad \text{McCumber parameter}$$

$$\frac{d^2 \delta}{dz^2} + \frac{1}{Q} \frac{d\delta}{dz} + \sin(\delta) = \frac{I}{I_0} \quad Q \propto \sqrt{I_0 C R}$$

pendulum with friction



homework:

- derive equation of motion
- identify terms

torque: $\vec{T} = \vec{r} \wedge m\vec{g}$

$$T = -\sin(\varphi) \ell m g$$

angular momentum: $\vec{L} = \vec{r} \wedge \vec{v}$

$$L = m \ell^2 \dot{\varphi} = m \ell^2 \dot{\varphi}$$

$$T = \frac{d}{dt} L$$

$$-m g \ell \sin(\varphi) = m \ell^2 \ddot{\varphi}$$

$$\ddot{\varphi} \ell + g \sin(\varphi) = 0$$

$$\ddot{\varphi} + \omega_0^2 \sin(\varphi) = 0$$

$$\omega_0 = \sqrt{\frac{g}{\ell}}$$

assuming an external torque T_{ext}

and an angular friction $T_{\text{fr}} = -f_{\text{fr}} \cdot \dot{\varphi}$

$$T + T_{\text{ext}} + T_{\text{fr}} = \frac{d}{dt} L$$

$$T_{\text{ext}} = m \ell^2 \ddot{\varphi} + f_{\text{fr}} \dot{\varphi} + m \ell g \sin(\varphi)$$

comparison with RCSJ model

bias current capacitance Josephson current
resistance

tilted washboard model

particle of mass $m_p = \left(\frac{\phi_0}{2\pi}\right) C$ with

viscous drag force $f_{fr} = \left(\frac{\phi_0}{2\pi}\right) \frac{1}{R}$

moving in an effective potential

$$V(\delta) = -E_J \cos(\delta) - \frac{\phi_0}{2\pi} I \delta$$

slide
"tilted washboard potential"

$$m_p \ddot{\delta} + f_{fr} \dot{\delta} - \frac{\partial V}{\partial \delta} = 0$$

for $I < I_0$ static solution

$$\delta = \arcsin\left(\frac{I}{I_0}\right) \quad V=0$$

overdamped $Q \ll 1$

$$\dot{\delta} = \frac{2\pi I_0 R}{\phi_0} \left(\frac{I}{I_0} - \sin \delta \right)$$

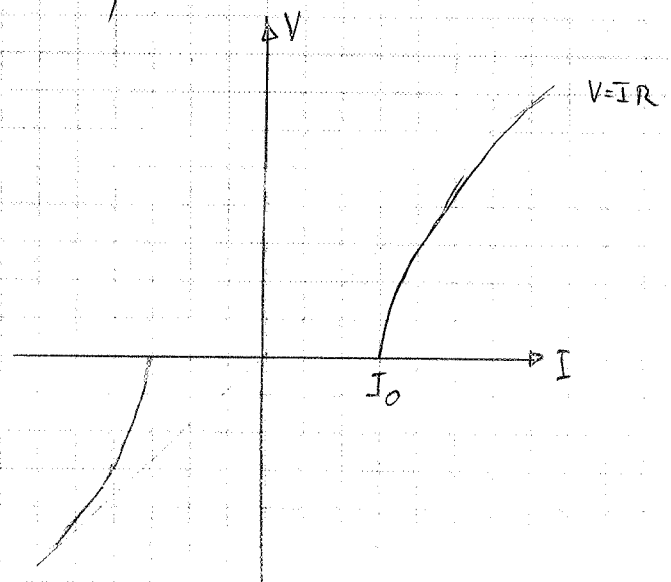
average voltage:

$$V = R \sqrt{I^2 - I_0^2}$$

which can be found
by integration over
one periode

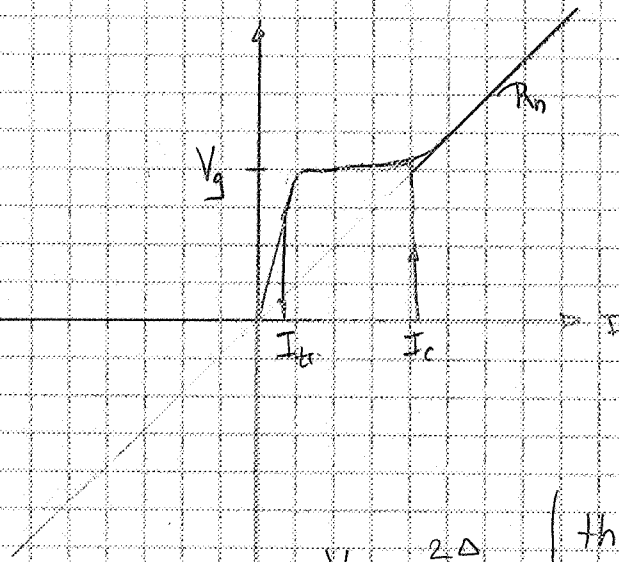
$$\int_0^{2\pi} \frac{d\delta}{\frac{I}{I_0} - \sin \delta} = \frac{2\pi I_0}{\phi_0} R \int_0^T dt$$

$$\frac{2\pi}{\sqrt{\left(\frac{I}{I_0}\right)^2 - 1}} = 2\pi \frac{I_0}{\phi_0} R T$$



$$V = \frac{\phi_0}{2\pi} \frac{1}{T} = R \sqrt{I^2 - I_0^2}$$

underdamped $Q > 1$
 hysteretic behaviour

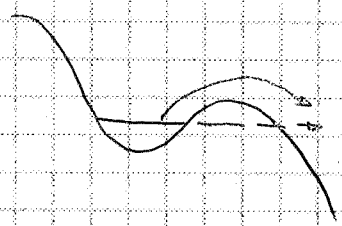


$V_g = \frac{2\Delta}{e}$ (this is not described with a constant R)

$I_{tr} = 4 I_0 \frac{1}{\pi Q}$

$I_c R_n = \pi \frac{\Delta}{2e}$

thermal fluctuations }
 quantum fluctuations } { reduce switching current
 increase retrapping current



thermal excitation over the barrier
 quantum tunneling through the barrier